1. Harvard Law School courses often have assigned seating to facilitate the “Socratic method.” Suppose that there are 100 first year Harvard Law students, and each takes two courses: Torts and Contracts. Both are held in the same lecture hall (which has 100 seats), and the seating is uniformly random and independent for the two courses.
2. Find the probability that no one has the same seat for both courses (exactly; you should leave your answer as a sum).   
   (b) Find a simple but accurate approximation to the probability that no one has the same seat for both courses.   
   (c) Find a simple but accurate approximation to the probability that at least two students have the same seat for both courses.

Sol)

(a) To find the probability that no one has the same seat for both courses, we can use the principle of inclusion-exclusion.

Number of cases where each specific student has the same seat for both courses:

For each student, there is only one seat out of 100 that matches their seat in the other course. So, there are 100 possibilities for the first student, 1 possibility for the second student (since their seat is already fixed), and so on. Therefore, the total number of cases where each specific student has the same seat for both courses is 1 \* 1 \* ... \* 1 = 1.

Number of cases where more than one student has the same seat for both courses:

If two students have the same seat for both courses, there are 100 possibilities for the first student to choose a seat, and only 1 possibility for the second student since their seat is fixed. Therefore, the total number of cases where two students have the same seat for both courses is 100 \* 1 = 100.

Since these cases are mutually exclusive, we can find the total number of cases where at least one student has the same seat for both courses by summing the two cases: 1 + 100 = 101.

The probability that no one has the same seat for both courses is then given by:

P(no same seat) = 1 - P(same seat)

P(no same seat) = 1 - (101/100!)

(b) To find a simple but accurate approximation to the probability, we can use the fact that for large values of n, n! can be approximated using Stirling's approximation.

Stirling's approximation: n! ≈ √(2πn)(n/e)^n

Using this approximation, we can rewrite the probability as:

P(no same seat) ≈ 1 - (101/√(2πn)(n/e)^n)

(c) To find a simple but accurate approximation to the probability that at least two students have the same seat for both courses, we can use the principle of complement.

The probability that at least two students have the same seat for both courses is equal to 1 minus the probability that no one has the same seat and the probability that exactly one student has the same seat.

P(at least two same seat) = 1 - P(no same seat) - P(one same seat)

1. There are 100 passengers lined up to board an airplane with 100 seats (with each seat assigned to one of the passengers). The first passenger in line crazily decides to sit in a randomly chosen seat (with all seats equally likely). Each subsequent passenger takes his or her assigned seat if available, and otherwise sits in a random available seat. What is the probability that the last passenger in line gets to sit in his or her assigned seat?

Sol)

The probability that the last passenger in line gets to sit in his or her assigned seat can be calculated as 1/2.